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SPATIAL ANALOG OF CENTERED RIEMANN AND PRANDTL–MEYER WAVES

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In this paper, we prove the existence of solutions of equations of spatial gasdynamics that have special properties: waves, centered on arbitrary two-dimensional surfaces in four-dimensional space **x**, t. These solutions are generalizations of the centered Riemann waves in the theory of one-dimensional nonstationary motion and centered Prandtl–Meyer waves in the theory of planar stationary flows. Characteristics of this form arise in problems of the interaction of shock waves with fronts having arbitrary shapes, interaction of shock waves and a contact discontinuity, and piston problems.

1. Formulation of the Problem. We are examining equations that describe the spatial instability of flow of a nonviscous, nonthermally conducting ordinary gas [1, 2]:

$$\frac{d\mathbf{u}}{dt}+\frac{1}{\rho}\nabla p=0,\ \frac{dp}{dt}+\rho c^2\operatorname{div}\mathbf{u}=0,\ \frac{dS}{dt}=0,\ \rho=\psi(p, S), \tag{1.1}$$

where **u** is the velocity vector; p, pressure; ρ, density; S, entropy; c, velocity of sound; t, time; $\mathbf{x} = (x, y, z)$, radius vector of a point in R^3; $\nabla = (\partial/\partial x, \partial/\partial y, \partial/\partial z)$; $d/dt = \partial/\partial t + \mathbf{u}\cdot\nabla$. The function ψ(p, S), which gives the equation of state of the ordinary gas, is assumed to be analytic.

A centered wave is a solution of the system (1.1) whose domain is covered by a single parameter family of acoustic characteristics passing through the given two-dimensional surface $\gamma_0 \subset E^4 = R^3 \times R$ ($\mathbf{x} \in R^3$, $t \in R$). In this case, the wave is said to be centered on γ_0.

In what follows, we examine the problem of a piston. Assume that the solution of system (1.1), satisfying the impermeability condition $\mathbf{u}\cdot\nabla h = 0$ on Γ is given in a half space, whose boundary Γ is given by the equation $h(\mathbf{x}) = 0$ ($\nabla h \neq 0$), is determined for $0 \leqslant t \leqslant t_0$ This solution in what follows is called the unperturbed solution. A perturbation propagating along Γ arises at time t = 0 at the point $Q \in \Gamma$: the lateral wall begins to buckle according to a definite law so that outside the buckled part, it is given by the equation $h(\mathbf{x}) = 0$, while in the buckled part Γ', it is given by equation $h_1(\mathbf{x}, t) = 0$. It is assumed that $h_1 > 0$ in the region occupied by the gas, $h_{1t} > 0$ on Γ', and the surfaces $h(\mathbf{x}) = 0$ and $h_1(\mathbf{x}, 0) = 0$ are tangent at the point Q. The intersection of Γ and Γ' forms an edge which moves according to a given law along Γ. Let γ_0 be a two-dimensional surface and E^4, traced out by this edge in time (Fig. 1 shows a picture illustrating the two-dimensional case). The unperturbed solution will describe a gas flow in the region bounded by the acoustic characteristic Γ_1 ($\varphi(\mathbf{x}, t) = 0$)

$$\varphi_t + \mathbf{u}\cdot\nabla\varphi + c|\nabla\varphi| = 0, \tag{1.2}$$

passing through γ_0 (φ > 0 in the region of unperturbed motion). It is necessary to find the

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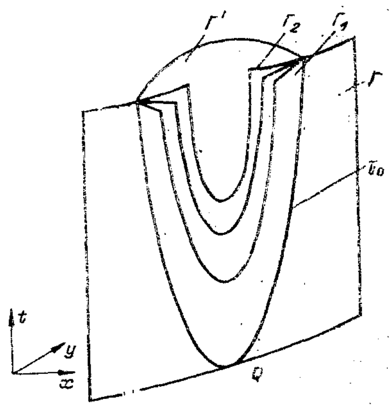

Fig. 1.

perturbed solution in the region bounded by Γ_1 and Γ', continuously adjacent along Γ_1 to the unperturbed solution, satisfying the impermeability condition on Γ':

$$h_{1t} + \mathbf{u}\cdot\nabla h_1 = 0. \tag{1.3}$$

Due to the fact that the matching conditions for data are not satisfied on Γ' and Γ_1, a centered wave arises on γ_0. Its domain is bounded by the acoustic characteristics Γ_1 and Γ_2, passing through γ_0. The limiting value of $\mathbf{u}$ on γ_0 along Γ_2 will satisfy (1.3). After finding the centered wave, it remains to solve the mixed problem without singularities with data on Γ', Γ_2.

It is well known that a stationary centered Prandtl–Meyer wave is a supersonic flow. Here, the analog of this fact is that the centered wave can be constructed in the vicinity of those points γ_0 which move along Γ' relative to gas with supersonic velocity in a direction normal to the cross section γ_{0t} of the surface γ_0 by the surface $t = \text{const}$. The coincidence of the velocity indicated with the velocity of sound corresponds to the appearance of points on γ_0, in which the characteristic strips of Eq. (1.2) are tangent to γ_0.

2. Transformation of the Equations. In the centered wave region, new independent variables τ, ξ, β, γ will be introduced with the help of the substitution $\mathbf{x} = \mathbf{x}(\tau, \xi, \beta, \gamma)$, $t = t(\tau, \beta, \gamma)$. The functions $\mathbf{x}(\tau, \xi, \beta, \gamma)$, $t(\tau, \beta, \gamma)$ are defined as the solutions of the Cauchy problem:

$$\mathbf{x}_\tau = \mathbf{f},\ t_\tau = 1,\ \mathbf{x}|_{\tau=0} = \mathbf{x}_0(\beta, \gamma),\ t|_{\tau=0} = t_0(\beta, \gamma). \tag{2.1}$$

The equations $\mathbf{x} = \mathbf{x}_0(\beta, \gamma)$, $t = t_0(\beta, \gamma)$ parametrically define γ_0; it is assumed that this mapping is mutually unique on γ_0, $|\mathbf{x}_{0\beta}| \neq 0$, $|\mathbf{x}_{0\gamma}| \neq 0$, $|\mathbf{x}_{0\beta} \times \mathbf{x}_{0\gamma}| \neq 0$ (here and in what follows, $\mathbf{a} \times \mathbf{b}$ is the vector product of $\mathbf{a}$ and $\mathbf{b}$). At the point Q, where γ_0 is tangent to the plane $t = 0$, $t_{0\beta} = t_{0\gamma} = 0$. The function $\mathbf{f}$ is chosen so that for fixed ξ the equations $\mathbf{x} = \mathbf{x}(\tau, \xi, \beta, \gamma)$, $t = t(\tau, \beta, \gamma)$ give the acoustic characteristic passing through γ_0 ($0 \leqslant \xi \leqslant 1$, $\xi = 0$ corresponds to Γ_1, and $\xi = 1$ corresponds to Γ_2). In view of (1.2), this requirement gives the relation

$$(\mathbf{f} - \mathbf{u})\cdot((\mathbf{x}_\beta - t_\beta\mathbf{f}) \times (\mathbf{x}_\gamma - t_\gamma\mathbf{f})) = c|(\mathbf{x}_\beta - t_\beta\mathbf{f}) \times (\mathbf{x}_\gamma - t_\gamma\mathbf{f})|,$$

which will be satisfied if $\mathbf{f}$ is taken in the form

$$\mathbf{f} = \mathbf{u} + c(|\mathbf{v}|^2 + 2i|\mathbf{k}|^2)|\mathbf{v}|^{-2}(|\mathbf{v}|^2 - |\mathbf{k}|^2c^2)^{-1/2}\cdot\mathbf{v} - 2i|\mathbf{v}|^{-2}(\mathbf{k} \times \mathbf{v}), \tag{2.2}$$

where $\mathbf{v} = (\mathbf{x}_\beta - t_\beta\mathbf{u}) \times (\mathbf{x}_\gamma - t_\gamma\mathbf{u})$; $\mathbf{k} = t_\beta\mathbf{x}_\gamma - t_\gamma\mathbf{x}_\beta$; $i = \varepsilon + p\rho^{-1}$; is the specific internal energy of the gas. Expression (2.2) is meaningful for $|\mathbf{v}| > c|\mathbf{k}|$. The centered wave will be constructed in the vicinity of those points γ_0, where the unperturbed solution satisfies the inequality 1°: $1 - |\mathbf{k}|c|\mathbf{v}|^{-1} \geqslant \sigma_1 > 0$ ($\sigma_1 = \text{const}$). In view of the fact that at the points Q, $t_\beta = t_\gamma = 0$ and therefore, $\mathbf{k} = 0$, the required inequality is always satisfied in the vicinity of the point Q. The equality $|\mathbf{v}| = |\mathbf{k}|c$ corresponds to the line γ_{0t} moving at the velocity of sound along the wall relative to the gas in a direction normal to it. For large t, the centered wave disappears and another singularity appears on γ_0. According to (2.1), the substitution of variables is degenerate on γ_0, since $\mathbf{x}_\xi(0, \xi, \beta, \gamma) = 0$. Let us represent $\mathbf{x}_\xi$ in the form $\mathbf{x}_\xi = \tau\mathbf{y}$. In substituting variables in (1.1), the equations

$$\frac{\partial}{\partial\tau} = \frac{\partial}{\partial t} + \mathbf{f}\cdot\nabla,\ \frac{\partial}{\partial\xi} = \tau\mathbf{y}\cdot\nabla,\ \frac{\partial}{\partial\beta} = t_\beta\frac{\partial}{\partial t} + \mathbf{x}_\beta\cdot\nabla,\ \frac{\partial}{\partial\gamma} = t_\gamma\frac{\partial}{\partial t} + \mathbf{x}_\gamma\cdot\nabla$$

and the resulting relations

$$d/dt = \partial/\partial\tau + (\mathbf{u} - \mathbf{f})\cdot\nabla, \quad \nabla = J^{-1}\{[(\mathbf{x}_\gamma - t_\gamma\mathbf{f}) \times \mathbf{y}]\partial/\partial\beta + (\mathbf{y} \times \mathbf{k})\partial/\partial\tau + [\mathbf{y} \times (\mathbf{x}_\beta - t_\beta\mathbf{f})]\partial/\partial\gamma + \tau^{-1}[(\mathbf{x}_\beta - t_\beta\mathbf{f}) \times (\mathbf{x}_\gamma - t_\gamma\mathbf{f})]\partial/\partial\xi\},$$

where $J = \mathbf{y}[(\mathbf{x}_\beta - t_\beta\mathbf{f}) \times (\mathbf{x}_\gamma - t_\gamma\mathbf{f})]$, are used. After some transformations, (1.1) is written in the form

$$\begin{gathered}(\mathbf{x}_\beta - t_\beta\mathbf{u})\mathbf{u}_\xi - t_\beta\rho^{-1}p_\xi - \tau[(\mathbf{y}\cdot\mathbf{v})(|\mathbf{v}|^2 - |\mathbf{k}|^2c^2)^{1/2}(|\mathbf{v}|^2c + \\ + 2ic|\mathbf{k}|^2)^{-1}\cdot((\mathbf{x}_\beta - t_\beta\mathbf{u})\mathbf{u}_\tau - t_\beta\rho^{-1}p_\tau) + A_1\mathbf{U}_\beta + A_2\mathbf{U}_\gamma] = 0, \\ (\mathbf{x}_\gamma - t_\gamma\mathbf{u})\mathbf{u}_\xi - t_\gamma\rho^{-1}p_\xi - \tau[(\mathbf{y}\cdot\mathbf{v})(|\mathbf{v}|^2 - |\mathbf{k}|^2c^2)^{1/2}(|\mathbf{v}|^2c + \\ + 2ic|\mathbf{k}|^2)^{-1}((\mathbf{x}_\gamma - t_\gamma\mathbf{u})\mathbf{u}_\tau - t_\gamma\rho^{-1}p_\tau) + B_1\mathbf{U}_\beta + B_2\mathbf{U}_\gamma] = 0, \\ S_\xi - \tau[(\mathbf{y}\cdot\mathbf{v})(|\mathbf{v}|^2 - |\mathbf{k}|^2c^2)^{1/2}(|\mathbf{v}|^2c + 2ic|\mathbf{k}|^2)^{-1}S_\tau + D_1S_\beta + D_2S_\gamma] = 0, \\ \mathbf{v}\cdot\mathbf{u}_\tau + (|\mathbf{v}|^2 - |\mathbf{k}|^2c^2)^{1/2}\rho^{-1}c^{-1}p_\tau + E_1\mathbf{U}_\beta + E_2\mathbf{U}_\gamma = 0, \\ \mathbf{v}\cdot\mathbf{u}_\xi - (|\mathbf{v}|^2 - |\mathbf{k}|^2c^2)^{1/2}\rho^{-1}c^{-1}p_\xi - \tau[((\mathbf{y}\cdot\mathbf{v})^2 - c^2(\mathbf{y} \times \mathbf{k})^2)(2cJ)^{-1}(|\mathbf{v}|^2 - \\ - |\mathbf{k}|^2c^2)^{-1/2}(\mathbf{v}\cdot\mathbf{u}_\tau - (|\mathbf{v}|^2 - |\mathbf{k}|^2c^2)^{1/2}\rho^{-1}c^{-1}p_\tau) + F_1\mathbf{U}_\beta + F_2\mathbf{U}_\gamma] = 0.\end{gathered} \tag{2.3}$$

Here, $\mathbf{U}$ indicates the vector solution whose components are p, S, and the components of $\mathbf{u}$; A_i, B_i, E_i, F_i are vector and D_i are scalar functions of the variables $\mathbf{U}$, $\mathbf{x}_\beta$, $\mathbf{x}_\gamma$, $\mathbf{y}$. For $\tau = 0$, it follows from (2.3) that the quantities S, $a = (\mathbf{x}_\beta\cdot\mathbf{u}) - t_\beta(2^{-1}|\mathbf{u}|^2 + i)$, $b = (\mathbf{x}_\gamma\cdot\mathbf{u}) - t_\gamma(2^{-1}|\mathbf{u}|^2 + i)$ are conserved when passing through the centered wave at the fixed point γ_0 ($S_\xi = a_\xi = b_\xi = 0$ with $\tau = 0$). Let us introduce the quantities θ and H:

$$\operatorname{tg}|\mathbf{k}|\theta = \frac{|\mathbf{k}|(\mathbf{u}\cdot\mathbf{m})}{(\mathbf{v}\cdot\mathbf{m})}, \quad \mathbf{m} = \mathbf{x}_\beta \times \mathbf{x}_\gamma, \quad H(p, S, B) = \int_0^p \frac{[B - |\mathbf{k}|^2(c^2 + 2i)(p', S)]^{1/2}\,dp'}{\rho(p', S)\,c(p', S)\,(B - 2|\mathbf{k}|^2 i(p', S))}$$

and $r = \theta + H(p, S, |\mathbf{v}|^2 + 2|\mathbf{k}|^2 i)$, $l = \theta - H(p, S, |\mathbf{v}|^2 + 2|\mathbf{k}|^2 i)$. From the last equation (2.3), it follows that for $\tau = 0$ $l_\xi = 0$, (in view of the first equations $(|\mathbf{v}|^2 + 2i|\mathbf{k}|^2)_\xi = 0$ at $\tau = 0$). It is evident that the change in $|\mathbf{k}|\theta$ characterizes the angle of rotation of the vector $\mathbf{v}$ in a plane orthogonal to $\mathbf{k}$, when passing through the centered wave. For $|\mathbf{k}| \to 0$, $\theta \pm H \to |\mathbf{m}|^{-1} \times \left(u_n \pm \int_0^p \rho^{-1}c^{-1}dp\right)$, where $u_n = (\mathbf{u}\cdot\mathbf{m})|\mathbf{m}|^{-1}$. If at a point on the surface γ_0 with coordinates β, γ are given, then all the possible states θ_0, p_0, S_0, $|\mathbf{v}_0|^2 + 2i_0|\mathbf{k}|^2$, $\mathbf{k}$, obtained by a transition with the help of the centered wave, lie on the curve

$$\theta - H(p, S_0, |\mathbf{v}_0|^2 + 2i_0|\mathbf{k}|^2) = \theta_0 - H(p_0, S_0, |\mathbf{v}_0|^2 + 2i_0|\mathbf{k}|^2) \tag{2.4}$$

in the plane (θ, p). This curve is uniquely projected on the θ, p axes. Giving either θ or p on γ_0 on the other side of the wave permits determining all quantities behind the wave. In the piston problem, $\theta_0 = 0$, $\lim_{\tau\to 0}(\mathbf{u}\cdot\mathbf{n})|_{\Gamma_2} = (-h_{1t}\cdot|\nabla h_1|^{-1})_{\Gamma_2,\tau=0}$, where $\mathbf{n} = \nabla h_1\cdot|\nabla h_1|^{-1}$, is given on γ_0. Since Γ' is the contact characteristic, $\mathbf{n} = \mathbf{v}\cdot|\mathbf{v}|^{-1}$. Then $(\mathbf{u}\cdot\mathbf{n}) = (\mathbf{u}\cdot\mathbf{m})\cdot|\mathbf{v}|^{-1} = |\mathbf{m}|\,|\mathbf{k}|^{-1}\sin|\mathbf{k}|\theta$. Therefore, $\lim_{\tau\to 0}\theta|_{\Gamma_2} = \theta_2(\beta, \gamma)$ is given on γ_0. The existence of a centered wave will be proved when the following restrictions 2° are satisfied:

$$\sigma_3 - H(p_0, S_0, |\mathbf{v}_0|^2 + 2i_0|\mathbf{k}|^2) \leqslant \theta_2(\beta, \gamma) \leqslant -\sigma_2,$$

where σ_2, σ_3 are positive constants. This inequality guarantees a nonzero wave amplitude ($\theta_2 \leqslant -\sigma_2$) and absence of a vacuum $\left(\lim_{\tau\to 0} p|_{\Gamma_2} > 0\right)$.

Let the unperturbed solution, the surfaces Γ, Γ' and γ_0 be analytic and inequalities 1°, 2° be satisfied.

THEOREM. There exists a centered wave analytic for $0 < \tau \leqslant \tau_0$ ($\tau_0 > 0$) in the vicinity of γ_0, continuously touching the unperturbed solution along the acoustic characteristic Γ_1 such that the limiting value of θ along Γ_2 with $\tau \to 0$ coincides with the given analytic function $\theta_2(\beta, \gamma)$.

3. Existence of a Solution in the Class of Formal Power Series. The quantities r, l, a, b, S, **x**, **y**, $\mathbf{z} = \mathbf{x}_\beta$, $\mathbf{w} = \mathbf{x}_\gamma$ are the functions sought. The value of $\theta_2(\beta, \gamma)$, according to Sec. 2, is determined by $r_2(\beta, \gamma)$, the limiting value of r along Γ_2 with $\tau = 0$. We set

$$r|_{\tau=0} = (1 - \xi)r_0(\beta, \gamma) + \xi r_2(\beta, \gamma), \tag{3.1}$$

where $r_0(\beta, \gamma)$ is the value of r at $\tau = \xi = 0$, which is known from the conditions of continuous touching. The conditions for continuous touching on Γ_1 have the form

$$l|_{\xi=0} = l_0(\tau, \beta, \gamma),\ a|_{\xi=0} = a_0(\tau, \beta, \gamma),\ b|_{\xi=0} = b_0(\tau, \beta, \gamma),\ S|_{\xi=0} = S_0(\tau, \beta, \gamma), \quad (3.2)$$

where l_0, a_0, b_0, S_0 are given analytic functions. We shall reduce the boundary conditions to homogeneous conditions, by replacing the functions sought: $l_1 = l - l_0$, $a_1 = a - a_0$, $b_1 = b - b_0$, $S_1 = S - S_0$, $\mathbf{x}_1 = \mathbf{x} - \mathbf{x}_0$, $\mathbf{w}_1 = \mathbf{w} - \mathbf{x}_{0\gamma}$, $\mathbf{z}_1 = \mathbf{z} - \mathbf{x}_{0\beta}$, $r_1 = r - (1 - \xi)r_0(\beta, \gamma) - \xi r_2(\beta, \gamma)$. In view of (2.1) and (2.3), we can compute $\mathbf{y}_0(\xi, \beta, \gamma) = \mathbf{y}|_{\tau=0} = (|\mathbf{v}|^2 + 2i|\mathbf{k}|^2)(|\mathbf{v}|^2 - |\mathbf{k}|^2c^2)^{-3/2}(\rho^{-1}c^{-1}p_\xi + c_\xi)\mathbf{v}|_{\tau=0}$. Let us set $\mathbf{y}_1 = \mathbf{y} - \mathbf{y}_0$. We now have a problem with homogeneous boundary conditions for the following system of equations:

$$\begin{gathered} a_{1\xi} = \tau[N_1 a_{1\tau} + A_{11}\mathbf{V}_\beta + A_{12}\mathbf{V}_\gamma + A_{13}\mathbf{y}_{1\beta} + A_{14}], \\ b_{1\xi} = \tau[N_1 b_{1\tau} + B_{11}\mathbf{V}_\beta + B_{12}\mathbf{V}_\gamma + B_{13}\mathbf{y}_{1\gamma} + B_{14}], \\ S_{1\xi} = \tau[N_1 S_{1\tau} + D_{11}\mathbf{V}_\beta + D_{12}\mathbf{V}_\gamma + D_{13}], \\ l_{1\xi} = \tau[N_2 l_{1\tau} + F_{11}S_{1\tau} + F_{12}a_{1\tau} + F_{13}b_{1\tau} + F_{14}\mathbf{V}_\beta \\ + F_{15}\mathbf{V}_\gamma + F_{16}\mathbf{y}_{1\beta} + F_{17}\mathbf{y}_{1\gamma} + F_{18}], \\ r_{1\tau} = E_{11}S_{1\tau} + E_{12}a_{1\tau} + E_{13}b_{1\tau} + E_{14}\mathbf{V}_\beta + E_{15}\mathbf{V}_\gamma + E_{16}, \\ \mathbf{x}_{1\tau} = \mathbf{G}_1,\ \mathbf{w}_{1\tau} = G_2\mathbf{V}_\beta + G_3\mathbf{V}_\gamma + \mathbf{G}_4,\ \mathbf{z}_{1\tau} = G_5\mathbf{V}_\beta + G_6\mathbf{V}_\gamma + \mathbf{G}_7, \\ \mathbf{y}_1 + \tau\mathbf{y}_{1\tau} = \mathbf{Q}_1 r_{1\xi} + \mathbf{Q}_2 + \tau[\mathbf{Q}_3 S_{1\tau} + \mathbf{Q}_4 a_{1\tau} + \mathbf{Q}_5 b_{1\tau} + \\ + \mathbf{Q}_6 l_{1\tau} + Q_7\mathbf{V}_\beta + Q_8\mathbf{V}_\gamma + Q_9\mathbf{y}_{1\beta} + Q_{10}\mathbf{y}_{1\gamma} + \mathbf{Q}_{11}], \\ \xi = 0:\ a_1 = b_1 = S_1 = l_1 = 0,\ \tau = 0:\ r_1 = 0,\ \mathbf{x}_1 = \mathbf{w}_1 = \mathbf{z}_1 = 0. \end{gathered} \quad (3.3)$$

Here, $\mathbf{V}$ is the vector solution with components a_1, b_1, S_1, l_1, r_1, $\mathbf{x}_1$, $\mathbf{w}_1$, $\mathbf{z}_1$; the coefficients A_{1i}, B_{1i}, D_{1i}, F_{1i}, E_{1i}, G_i, Q_i, N_i depend analytically on $\mathbf{V}$, $\mathbf{y}_1$, ξ, τ, β, γ. We note that $\mathbf{Q}_1$, $\mathbf{Q}_2$ do not depend on $\mathbf{y}_1$, $\mathbf{Q}_2|_{\tau=0} = 0$,

$$N_1 = \frac{(\mathbf{y}\cdot\mathbf{v})(|\mathbf{v}|^2 - |\mathbf{k}|^2 c^2)^{1/2}}{c(|\mathbf{v}|^2 + 2i|\mathbf{k}|^2)},\ N_2 = \frac{(\mathbf{y}\cdot\mathbf{v})^2 - c^2(\mathbf{y}\times\mathbf{k})^2}{2cJ(|\mathbf{v}|^2 - |\mathbf{k}|^2 c^2)^{1/2}}.$$

In order to construct a solution in the form of formal power series with the variables τ, $\xi - \xi_1$, $\beta - \beta_1$, $\gamma - \gamma_1$, where ξ_1, β_1, γ_1 are the coordinates of an arbitrary point in the plane $\tau = 0$, it is sufficient to calculate all derivative solutions at this point.

LEMMA. Derivative solutions are determined uniquely from the equations and the boundary conditions.

Proof. According to Sec. 2 (3.3), the functions sought vanish at $\tau = 0$. Let us assume that at $\tau = 0$ all derivative solutions of order $(j - 1)$ are known. Then, differentiating them with respect to ξ, β, γ, it is possible to find all derivatives of order j, except $\partial^j/\partial\tau^j$. The first four equations (3.3) can be represented in the form

$$\begin{gathered} a_{1\xi} - \tau L(\xi)a_{1\tau} = \tau\Phi_1,\ b_{1\xi} - \tau L(\xi)b_{1\tau} = \tau\Phi_2, \\ S_{1\xi} - \tau L(\xi)S_{1\tau} = \tau\Phi_3,\ l_{1\xi} - 2^{-1}\tau L(\xi)l_{1\tau} = \tau\Phi_4, \end{gathered} \quad (3.4)$$

where $L(\xi) = [(\rho^{-1}c^{-1}p_\xi + c_\xi)|\mathbf{v}|^2(c|\mathbf{v}|^2 - |\mathbf{k}|^2c^3)^{-1}]\ (0, \xi, \beta_1, \gamma_1) = N_1(0, \xi, \beta_1, \gamma_1) = 2N_2(0, \xi, \beta_1, \gamma_1)$. From (3.4), we obtain an ordinary differential equation for $\partial^j a_1/\partial\tau^j|_{\tau=0}$:

$$\left(\frac{\partial}{\partial\xi}\frac{\partial^j a_1}{\partial\tau^j} - jL(\xi)\frac{\partial^j a_1}{\partial\tau^j}\right)_{\tau=0} = j\frac{\partial^{j-1}}{\partial\tau^{j-1}}\Phi_1|_{\tau=0}.$$

Then,

$$\frac{\partial^j a_1}{\partial\tau^j}(0, \xi_1, \beta_1, \gamma_1) = j\int_0^{\xi_1}\frac{\partial^{j-1}}{\partial\tau^{j-1}}\Phi_1(0, \xi', \beta_1, \gamma_1)\exp\left(j\int_{\xi'}^{\xi_1}L(\xi'')\,d\xi''\right)d\xi'. \quad (3.5)$$

Analogous equations are obtained for $\partial^j b_1/\partial\tau^j$, $\partial^j S_1/\partial\tau^j$, $\partial^j l_1/\partial\tau^j$. These equations permit determining the derivatives indicated in terms of known quantities. The remaining derivatives are calculated from Eq. (3.3). The lemma is proved.

4. Majorant Problem. Here we shall indicate a problem whose solution will give the majorants for the previously found formal series in the vicinity of an arbitrary point $\tau = 0$, $\xi = \xi_1$, $\beta = \beta_1$, $\gamma = \gamma_1$. As in (3.5), we obtain an equation

$$\frac{\partial^{j+m+n} a_1(0, \xi_1, \beta_1, \gamma_1)}{\partial\tau^j \partial\beta^m \partial\gamma^n} = j \int_0^{\xi_1} \frac{\partial^{j-1+m+n}\Phi_1}{\partial\tau^{j-1}\partial\beta^m\partial\gamma^n}(0, \xi', \beta_1, \gamma_1) \exp\left(j \int_{\xi'}^{\xi_1} L(\xi'')\, d\xi''\right) d\xi'. \quad (4.1)$$

In view of the conditions 2° $L(\xi) < -\sigma_4$ $(\sigma_4 > 0)$. Then, from (4.1) follows the estimate

$$\left|\frac{\partial^{j+m+n} a_1}{\partial\tau^j\partial\beta^m\partial\gamma^n}(0, \xi_1, \beta_1, \gamma_1)\right| \leqslant \frac{1}{\sigma_4} \max_{\xi} \left|\frac{\partial^{j-1+m+n}\Phi_1}{\partial\tau^{j-1}\partial\beta^m\partial\gamma^n}(0, \xi, \beta_1, \gamma_1)\right|. \quad (4.2)$$

Analogous estimates for the derivatives b_1, S_1, l_1 are obtained from (3.4). The majorant system of equations is constructed as follows: the coefficients of equations (3.3) are replaced by their majorants uniform with respect to ξ, i.e., analytic functions, whose expansion coefficients in a power series in a neighborhood of the point $V = 0$, $\mathbf{y}_1 = 0$, $\beta = \beta_1$, $\gamma = \gamma_1$, $\tau = 0$, $\xi = \xi_1$ are not less than maxima with respect to ξ of the moduli of the corresponding expansion coefficients of the starting functions at the point $V = 0$, $\mathbf{y}_1 = 0$, $\beta = \beta_1$, $\gamma = \gamma_1$, $\tau = 0$, ξ $(0 \leqslant \xi \leqslant 1)$. The existence of such majorants for the coefficients of the system (3.3) can be established with the help of Cauchy's integral equations for analytic functions of many variables. In addition to these equations, we examine one more group of equations:

$$\frac{\partial a_2}{\partial\tau} = \sigma_4^{-1}\Phi_{21}, \ \frac{\partial b_2}{\partial\tau} = \sigma_4^{-1}\Phi_{22}, \ \frac{\partial S_2}{\partial\tau} = \sigma_4^{-1}\Phi_{23}, \ \frac{\partial l_2}{\partial\tau} = 2\sigma_4^{-1}\Phi_{24}. \quad (4.3)$$

In these equations, the right sides Φ_{2i} are constructed according to Φ_i as follows: in Φ_i, a_1, b_1, S_1 and l_1 are replaced by a_2, b_2, S_2 and l_2, and then the coefficients in front of the derivatives are replaced by their uniform, with respect to ξ, majorants as in the preceding case. The boundary conditions of the majorant problem are:

$$\tau = 0\colon r_{1m} = a_2 = b_2 = S_2 = l_2 = 0, \ \mathbf{x}_{1m} = \mathbf{w}_{1m} = \mathbf{z}_{1m} = 0;$$
$$\xi = \xi_1\colon a_{1m} - a_2 = b_{1m} - b_2 = S_{1m} - S_2 = l_{1m} - l_2 = 0.$$

Here, r_{1m}, a_{1m}, and so on are majorants of r_1, a_1, and so on.

All the derivatives of the solution of the majorant problem are determined uniquely. From the method for constructing the majorant equations and (4.1)-(4.3), it follows that these derivatives are not less than maxima with respect to ξ of moduli of the derivative solutions of the starting problem, calculated with $\tau = \beta - \beta_1 = \gamma - \gamma_1 = 0$, $0 \leqslant \xi \leqslant 1$. The majorant problem can be simplified. We shall reduce the boundary conditions to homogeneous conditions by the substitution $a_3 = a_{1m} - a_2$, $b_3 = b_{1m} - b_2$, $S_3 = S_{1m} - S_2$, $l_3 = l_{1m} - l_2$. Then, Eqs. (4.3) can be solved for $a_{2\tau}$, $b_{2\tau}$, $S_{2\tau}$, $l_{2\tau}$, since these derivatives enter on the right sides with a coefficient that vanishes at $\beta - \beta_1 = \gamma - \gamma_1 = 0$. Groups of similar equations are replaced by a single equation, introducing the overall majorant A for functions a_2, b_2, S_2, l_2, overall majorant B for functions a_3, b_3, S_3, l_3, overall majorant P for functions r_{1m}, $\mathbf{x}_{1m}$, $\mathbf{w}_{1m}$, $\mathbf{z}_{1m}$, overall majorant Y for functions $\mathbf{y}_{1m}$. Equations for the overall majorants are obtained by summing the equations for a group of one type of equation. Independent variables enter into the majorant equations as combinations of τ, $\zeta = \xi - \xi_1$, $\delta = (\beta - \beta_1) + (\gamma - \gamma_1)$, so that the solution will be sought in the class of functions depending on τ, ζ, δ. The simplified majorant problem has the form

$$\begin{gathered} A_\tau = M_1 A_\delta + M_2 P_\delta + M_3 B_\delta + M_4 Y_\delta + M_5, \\ P_\tau = M_6 B_\tau + M_7 A_\delta + M_8 P_\delta + M_9 B_\delta + M_{10}, \\ B_\zeta = M_{11} A_\zeta + \tau(M_{12} B_\tau + M_{13} A_\delta + M_{14} P_\delta + M_{15} B_\delta + M_{16} Y_\delta + \\ + M_{17}), \ \tau Y_\tau + Y = M_{18} P_\zeta + M_{19} A + M_{20} P + M_{21} B + \tau(M_{22} B_\tau + \\ + M_{23} A_\delta + M_{24} P_\delta + M_{25} B_\delta + M_{26} Y_\delta + M_{27}), \end{gathered} \quad (4.4)$$

5. Invariant Majorants. In proving the existence of analytic majorants, a major difficulty arises due to the fact that the last equation in (4.4) with respect to Y at $\tau = 0$ is degenerate, as a result of which the methods applicable in Cauchy's problem and in a mixed problem are not applicable here.

The majorant M_i will be chosen so that Eqs. (4.4) will admit a nontrivial group of extensions of independent and dependent variables. The solution of system (4.4) will be found in a class of solutions invariant [3] relative to extensions. For any given analytic function Φ of the variables τ, ζ, δ, A, P, B, Y in the neighborhood of the point $\tau = \zeta = \delta = A = P = B = Y = 0$, a majorant of the following form can be found:

$$M = K[(1 - K\zeta)(1 - m_1\tau)(1 - m_2A)(1 - m_3B)(1 - m_4P)(1 - m_5Y)(1 - m_6\delta)]^{-1}$$

by choosing large enough constants K and m_j. Together with M, the function $M_i' = M_0(1 - K\zeta)^{-l_i}$, where

$$M_0 = K\left[\left(1 - m_1\tau(1 - K\zeta)^{-n_1}\right)\left(1 - m_2A(1 - K\zeta)^{-n_2}\right) \times \times\left(1 - m_3B(1 - K\zeta)^{-n_3}\right)\left(1 - m_4P(1 - K\zeta)^{-n_4}\right) \times \times\left(1 - m_5Y(1 - K\zeta)^{-n_5}\right)(1 - m_6\delta)\right]^{-1}$$

with integers $n_j \geqslant 0$ ($j = 1, \ldots, 5$), $l_i \geqslant 1$ will be a majorant of Φ. This follows from the fact that $(1 - K\zeta)^{-n} \gg 1$ with $n \gg 0$ ($\gg$ is the majorizing relation). Let us choose constants K and m_j such that the function M is a majorant of all M_i from (4.4). Now, if we examine a system of the form (4.4) with $M_i' = M_0(1 - K\zeta)^{-l_i}$ for $i \neq 11, 18, 19, 20, 21$, $M_{11}' = (1 - K\zeta)^{-l_{11}}$ $M_i' = M_{00}(1 - K\zeta)^{-l_i}$ for $i = 18, 19, 21$, and $M_{20}' = 2KM_{00}(1 - K\zeta)^{-l_{20}}$ ($M_{00} = M_0|_{m_5=0}$) with $n_j \geqslant 0$, $l_i \geqslant 1$ as yet undetermined, then this system also is a majorant for the starting problem. The choice of n_j, l_i is made in such a way that the system (4.4) admits a stretching transformation:

$$(1 - K\zeta) \to \varkappa(1 - K\zeta), \quad \tau \to \varkappa^{n_1}\tau, \quad A \to \varkappa^{n_2}A, \quad B \to \varkappa^{n_3}B,$$
$$P \to \varkappa^{n_4}P, \quad Y \to \varkappa^{n_5}Y, \quad \delta \to \delta,$$

where $\varkappa$ is the stretching parameter. The condition of invariance of (4.4) leads to a linear homogeneous system of equations for the indices l_i with a number of equations that is less than the number of unknowns. One of the solutions has the form $l_4 = l_5 = l_6 = l_{11} = l_{12} = l_{18} = 1$, $l_{20} = 2$, $l_2 = l_{10} = l_{16} = l_{17} = l_{21} = l_{22} = 3$, $l_3 = l_{19} = 4$, $l_1 = l_8 = l_{14} = l_{26} = l_{27} = 5$, $l_9 = l_{15} = 6$, $l_7 = l_{13} = l_{24} = 7$, $l_{25} = 8$, $l_{23} = 9$. In this case $n_1 = 5$, $n_2 = 4$, $n_3 = 3$, $n_4 = 2$, $n_5 = 0$. The invariant, relative to the stretching as indicated, solution is sought in the form

$$A = (1 - K\zeta)^4A_0(\eta, \delta),\ B = (1 - K\zeta)^3B_0(\eta, \delta),\ P = (1 - K\zeta)^2P_0(\eta, \delta),\ Y = Y(\eta, \delta),\ \eta = \tau(1 - K\zeta)^{-5}.$$

From (4.4), we obtain a system for determining the functions A_0, B_0, P_0, Y:

$$A_{0\eta} = M_0(A_{0\delta} + P_{0\delta} + B_{0\delta} + Y_\delta + 1),\ P_{0\eta} = M_0(B_{0\eta} + A_{0\delta} + P_{0\delta} + B_{0\delta} + 1),$$
$$4K\eta B_{0\eta} - 3KB_0 = 5K\eta A_{0\eta} - 4KA_0 + + (M_0 - K)\eta B_{0\eta} + M_0\eta(A_{0\delta} + P_{0\delta} + B_{0\delta} + Y_\delta + 1),$$
$$\eta Y_\eta + Y = M_{00}(5K\eta P_{0\eta} + A_0 + B_0) + \eta M_0(B_{0\eta} + A_{0\delta} + P_{0\delta} + B_{0\delta} + Y_\delta + 1). \qquad (5.1)$$

We note that $M_0 - K \gg 0$, i.e., it is a majorant type function. We seek a solution of system (5.1) that vanishes at $\eta = 0$. From the last two equations in (5.1), we obtain equations for determining the derivatives of B_0 and Y at $\eta = 0$:

$$\frac{\partial^{n+m}B_0}{\partial\eta^n\partial\delta^m} = \frac{5n-4}{4n-3}\frac{\partial^{n+m}A_0}{\partial\eta^n\partial\delta^m} + \frac{n}{4n-3}\frac{\partial^{n-1+m}}{\partial\eta^{n-1}\partial\delta^m}[(M_0 - K)B_{0\eta} + 4M_0(A_{0\delta} + P_{0\delta} + B_{0\delta} + Y_\delta + 1)],$$
$$\frac{\partial^{n+m}Y}{\partial\eta^n\partial\delta^m} = \frac{n}{n+1}\frac{\partial^{n-1+m}}{\partial\eta^{n-1}\partial\delta^m}\left[5KM_{00}P_{0\eta} + \frac{1}{n}\frac{\partial}{\partial\eta}M_{00}(A_0 + B_0) + M_0(B_{0\eta} + A_{0\delta} + P_{0\delta} + B_{0\delta} + Y_\delta + 1)\right].$$

It follows from these equations that the derivatives determined are nonnegative, while the majorants, with respect to the last two equations in (5.1), are the equations:

$$B_{0\eta} = (5/4)A_{0\eta} + (M_0 - K)B_{0\eta} + 4M_0(A_{0\delta} + P_{0\delta} + B_{0\delta} + Y_\delta + 1),$$
$$Y_\eta = 5KM_{00}P_\eta + \frac{\partial}{\partial\eta}(M_{00}(A_0 + B_0)) + M_0(B_{0\eta} + A_{0\delta} + P_{0\delta} + B_{0\delta} + Y_\delta + 1). \qquad (5.2)$$

The system formed by Eqs. (5.2) and the first two equations in (5.1) can be solved for the derivatives with respect to η ($M_0 - K$ vanishes at $\eta = \zeta = \delta = A_0 = B_0 = P_0 = Y = 0$). According to the Cauchy–Kovalevsky theorem, this system has an analytic solution that vanishes at $\eta = 0$. Therefore, the existence of an analytic solution of the majorant system such that $A = P = B = 0$ at $\tau = 0$, while $B|_{\zeta=0} \gg 0$ has been proved (since $B_0 = 0$ at $\eta = 0$, $B_0 = \eta B_{00}(\eta, \delta)$, where B_{00} is a majorant type function, $B = \tau(1 - K\zeta)^{-2}B_{00}(\eta, \delta) \gg 0$). The existence of an analytic solution of the problem (3.3) is thereby proved.

6. Transformation to the Variables **x**, t. In this section, we shall prove the single-sheet nature of the mapping $\mathbf{x} = \mathbf{x}(\tau, \xi, \beta, \gamma)$, $t = t(\tau, \beta, \gamma)$ with small τ ($\tau \neq 0$) in the

vicinity of γ_0. This makes it possible to invert the mapping indicated and to obtain a solution of the problem in the variables $\mathbf{x}$, t, analytic in this neighborhood, with the exception of the point γ_0. We shall prove that for small τ, from the equalities $\mathbf{x}(\tau_1, \xi_1, \beta_1, \gamma_1) = \mathbf{x}(\tau_2, \xi_2, \beta_2, \gamma_2)$, $\tau_1 + t_0(\beta_1, \gamma_1) = \tau_2 + t_0(\beta_2, \gamma_2)$, follow the equalities $\tau_1 - \tau_2 = \xi_1 - \xi_2 = \beta_1 - \beta_2 = \gamma_1 - \gamma_2 = 0$. Let $\tau_1 \geqslant \tau_2$, $\tau_1 > 0$. In view of (2.1), the following equation is valid:

$$\mathbf{x}(\tau, \xi, \beta, \gamma) = \mathbf{x}_0(\beta, \gamma) + \int_0^\tau \mathbf{f}(\tau', \xi, \beta, \gamma)\, d\tau'. \tag{6.1}$$

In a neighborhood of the point γ_0, where $|\mathbf{f}| \leqslant K_1$ (K_1 is a positive constant), the inequality

$$|\mathbf{x}_0(\beta_2, \gamma_2) - \mathbf{x}_0(\beta_1, \gamma_1)| \leqslant K_1(\tau_1 + \tau_2).$$

is satisfied. In view of the equality $\tau_1 - \tau_2 = t_0(\beta_2, \gamma_2) - t_0(\beta_1, \gamma_1)$ and the properties of the given mapping $\mathbf{x} = \mathbf{x}_0(\beta, \gamma)$, $t = t_0(\beta, \gamma)$, a $K_2 > 0$ is found such that $|\beta_1 - \beta_2| + |\gamma_1 - \gamma_2| \leqslant K_2(\tau_1 + \tau_2)$. The equality $\mathbf{x}(\tau_1, \xi_1, \beta_1, \gamma_1) = \mathbf{x}(\tau_2, \xi_2, \beta_2, \gamma_2)$ in view of (6.1) is written in the form

$$\begin{aligned} &\mathbf{x}_0(\beta_1, \gamma_1) - \mathbf{x}_0(\beta_2, \gamma_2) + (t_0(\beta_2, \gamma_2) - t_0(\beta_1, \gamma_1))\,\mathbf{f}(0, \xi_2, \beta_2, \gamma_2) + \\ &\quad + \tau_1(\mathbf{f}(0, \xi_1, \beta_1, \gamma_1) - \mathbf{f}(0, \xi_2, \beta_2, \gamma_2)) \\ &= \int_0^{\tau_1} [\mathbf{f}(0, \xi_1, \beta_1, \gamma_1) - \mathbf{f}(\tau', \xi_1, \beta_1, \gamma_1) + \mathbf{f}(\tau', \xi_2, \beta_2, \gamma_2) - \\ &\quad - \mathbf{f}(0, \xi_2, \beta_2, \gamma_2)]\, d\tau' + \int_{\tau_1}^{\tau_2} [\mathbf{f}(\tau', \xi_2, \beta_2, \gamma_2) - \mathbf{f}(0, \xi_2, \beta_2, \gamma_2)]\, d\tau'. \end{aligned} \tag{6.2}$$

From this relation, we obtain the inequality

$$\begin{aligned} &|(\mathbf{x}_{0\beta}(\beta_2, \gamma_2) - t_{0\beta}(\beta_2, \gamma_2)\mathbf{f}(0, \xi_2, \beta_2, \gamma_2))(\beta_1 - \beta_2) + (\mathbf{x}_{0\gamma}(\beta_2, \gamma_2) - \\ &- t_{0\gamma}(\beta_2, \gamma_2)\,\mathbf{f}(0, \xi_2, \beta_2, \gamma_2)(\gamma_1 - \gamma_2)| \leqslant K_3\tau_1(|\xi_1 - \xi_2| + |\beta_1 - \beta_2| + |\gamma_1 - \gamma_2|), \end{aligned}$$

where the positive constant K_3 depends on K_2, maxima of the first derivatives of $\mathbf{f}$, second derivatives of $\mathbf{x}_0$, t_0 in the neighborhood examined. In view of the linear independence of the vectors $\mathbf{x}_\beta - t_\beta\mathbf{f}$, $\mathbf{x}_\gamma - t_\gamma\mathbf{f}$ at γ_0, we can find $\sigma_5 > 0$, $K_4 > 0$ such that for $0 < \tau_1 < \sigma_5$

$$|\beta_1 - \beta_2| + |\gamma_1 - \gamma_2| \leqslant K_4\tau_1|\xi_1 - \xi_2|. \tag{6.3}$$

Now, from relation (6.2), it follows that

$$\mathbf{x}_{0\beta}(\beta_2, \gamma_2)(\beta_1 - \beta_2) + \mathbf{x}_{0\gamma}(\beta_2, \gamma_2)(\gamma_1 - \gamma_2) + \tau_1\mathbf{f}_1 - \tau_2\mathbf{f}_2 = O(\tau_1^2|\xi_1 - \xi_2|), \tag{6.4}$$

where $\mathbf{f}_i = \mathbf{f}(0, \xi_i, \beta_2, \gamma_2)$. We introduce the analog to the Mach angle α: $\sin|\mathbf{k}|\alpha = |\mathbf{k}|c|\mathbf{v}|^{-1}$. Then from Eq. (2.2), we obtain the equations

$$|\mathbf{m}|^{-1}(\mathbf{f}\cdot\mathbf{m}) = \frac{|\mathbf{v}|^2 + 2i|\mathbf{k}|^2}{|\mathbf{v}|\cdot|\mathbf{k}|}\frac{\sin|\mathbf{k}|(\theta + \alpha)}{\cos|\mathbf{k}|\alpha}, \quad (\mathbf{f}\cdot(\mathbf{k}\times\mathbf{m})) = |\mathbf{m}|^2 - \frac{|\mathbf{v}|^2 + 2i|\mathbf{k}|^2}{|\mathbf{v}|}\frac{\cos|\mathbf{k}|(\theta + \alpha)}{\cos|\mathbf{k}|\alpha}\cdot|\mathbf{m}|.$$

Forming the scalar product of relation (6.4) with $\mathbf{m}$ and $\mathbf{k} \times \mathbf{m}$ and using the preceding equations, we obtain the equality

$$\frac{1}{|\mathbf{k}|}\frac{\sin|\mathbf{k}|(\theta_1 + \alpha_1)}{|\mathbf{v}_1|\cos|\mathbf{k}|\alpha_1} = \frac{\tau_2}{\tau_1}\frac{1}{|\mathbf{k}|}\frac{\sin|\mathbf{k}|(\theta_2 + \alpha_2)}{|\mathbf{v}_2|\cos|\mathbf{k}|\alpha_2} + O(\tau_1|\xi_1 - \xi_2|),$$

$$\frac{\cos|\mathbf{k}|(\theta_1 + \alpha_1)}{|\mathbf{v}_1|\cos|\mathbf{k}|\alpha_1} = \frac{\tau_2}{\tau_1}\frac{\cos|\mathbf{k}|(\theta_2 + \alpha_2)}{|\mathbf{v}_2|\cos|\mathbf{k}|\alpha_2} + O(\tau_1|\xi_1 - \xi_2|).$$

The indices 1 and 2 indicate the values of the corresponding functions at the points $(0, \xi_1, \beta_2, \gamma_2)$ and $(0, \xi_2, \beta_2, \gamma_2)$. From these equalities, follows the inequality

$$|\theta_1 + \alpha_1 - \theta_2 - \alpha_2| \leqslant K_5\tau_1|\xi_1 - \xi_2|,$$

where the positive constant K_5 is chosen uniformly with respect to $|\mathbf{k}|$. According to (3.1), r varies strictly monotonically with respect to ξ at $\tau = 0$. In view of the conditions of a normal gas, the same is valid for $\theta + \alpha$. For this reason, there exists a positive σ_6 such that $|\theta_1 + \alpha_1 - \theta_2 - \alpha_2| \geqslant \sigma_6|\xi_1 - \xi_2|$. For $0 < \tau_1 < \min(\sigma_5, \sigma_6K_5^{-1})$, it follows from the last inequalities that $\xi_1 = \xi_2$ and then, from (6.3) follow the equalities $\beta_1 - \beta_2 = \gamma_1 - \gamma_2 = 0$ and $\tau_1 = \tau_2$. The single-sheet nature of the mapping with $\tau \neq 0$ is proved.

This completes the proof of the theorem formulated in Sec. 2.

For a complete solution of the piston problem, it remains to construct a solution of the mixed problem for Eqs. (1.1) with the impermeability condition on Γ' and the condition of continuous touching to the centered wave on the characteristic Γ_2. Here the conditions for the consistency of the given boundary value problems are already satisfied. We note that the result obtained can also be used in problems of describing the interaction of strong discontinuities. The case when the surface γ_0 lies in the hyperplane $t = 0$ in E^4 is examined in [4]. Such centered waves arise in describing the decomposition of an arbitrary discontinuity on a curvilinear surface [5].

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INJECTION EFFECT IN A CONTAINED EXPLOSION IN A LIQUID-SATURATED MEDIUM

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The escape of gaseous products from the cavity of a contained explosion into dry rock has been considered [1, 2]. The escape is due to the residual elevated pressure in the cavity. However, a contained explosion in a liquid-saturated medium may result in an elevated pore pressure around the explosion cavity, which exceeds the pressure in the cavity itself. This is possible because the fluid in the pores is compressed when the shock wave passes, and the strength of the skeleton means that the pressure does not revert to the initial value on unloading: there remains a residual pressure in the pores of the order of the strength of the skeleton. On the other hand, the pressure in the explosion cavity at the end of the explosion is close to the lithostatic pressure, i.e., below the pressure in the pores. Therefore, the elevated pore pressure may cause implosion, namely injection of liquid into the cavity. This alters the temperature and pressure within the cavity, which in turn influences the cavity collapse.

Here we consider theoretically the implosion effect and the influence on the heat and mass transfer on explosion in a liquid-saturated medium.

Model for Heat and Mass Transfer after Explosion in a Water-Saturated Medium. It has been pointed out [3] that there may be a rise in the pore pressure after a contained explosion for the model of [4]. Figure 1 shows a typical graph for the pore pressure. The rise in pore pressure after the passage of a shock wave is indirectly confirmed by the ground water-level measurements after explosions [5]. Therefore, an explosion in a saturated rock may result in filtration not from the cavity but into it. When the liquid enters the cavity, where the temperature is about 10^4°K and the pressure about 15 MPa, the liquid evaporates, taking up energy from the rock vapor, which is thereby cooled. When that vapor reaches a state of saturation, it begins to condense and release latent heat. This may raise the

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